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The single tree-based stand simulator SILVA: construction, application and evaluation

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Abstract

Single tree forest growth simulators have proven to be very flexible tools for predicting forest growth. They are commonly applied in the United States but they are rarely used in European forest management yet. One of the few exceptions is the simulator SILVA, which is presented in this paper. The underlying equations of this simulator are developed from a set of more than 155,000 tree observations, including the species Norway spruce (*Picea abies* (L.) Karst.), Silver fir (*Abies alba* Mill.), Scots pine (*Pinus sylvestris* L.), Common beech (*Fagus sylvatica* L.) and Sessile oak (*Quercus petraea* (Mattuschka) Liebl.). These basic model equations are presented together with respective parameter values, derived from the aforesaid data set.

The benefits of SILVA for forest practice, research, and education are demonstrated with three exemplary applications. Evaluation in terms of biological plausibility, validation, and practical use of the implemented software is presented, which shows that: (1) biological principles could be represented without explicitly being implemented, and (2) the simulation results are very close to independent observations as far as the range of parameterized site conditions is not exceeded. Furthermore, possible implications of changing conditions or an application of the model across a wider geographical range are discussed with special reference to the ongoing process of parameterization. © 2002 Published by Elsevier Science B.V.

Keywords: Growth and yield; Stand growth simulator; Single tree-based modeling; Model application; Model evaluation; Validation; Implementation

1. Introduction

Single tree forest growth models represent a stand as a mosaic of single trees and simulate individual growth and interactions with or without consideration of tree position (Munro, 1974). The core of this kind of model is an equation system, which controls growth behavior of single trees depending on their relative position within the stand. Therefore, competition indices are derived, which characterize the growth condition of each tree, and which are used to predict its increment of height, diameter, etc. in the following period (Wensel and Koehler, 1985; Pukkala, 1989; Biging and Dobbertin, 1992; Bachmann, 1998). Results obtained with lower resolution models, e.g. stand-oriented growth models, which describe the stand only with mean values (Pienaar et al., 1990; Hui and Gadow, 1993), and models focused on stem number frequencies (Moser, 1974; Gadow, 1987), can be derived from single tree model output by summation (cf. Burkhart, 1987). Additionally, the higher resolution allows dealing with pure and mixed stands of principally every age structure and pattern, and provides a higher flexibility, particularly suitable to respond to new management purposes.

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The first single tree model was developed for pure Douglas fir stands (Newnham, 1964) and was followed by models for pure stands of other species (e.g. Mitchell, 1969; Bella, 1970; Arney, 1972; Mitchell, 1975). In the mid-1970s, the construction principles were applied to uneven-aged pure and mixed stands (Ek and Monserud, 1974; Monserud, 1975). Munro (1974) distinguishes between distance-dependent models, which use actual stem positions for calculating distances and competition, and distance-independent approaches. The worldwide bibliography of single tree growth models (Dudek and Ek, 1980) lists more than 40 different single tree models where both model types are approximately equally represented.

Since the 1980s single tree models have been widely used in the United States (Wykoff et al., 1982; Burkhart, 1987; Wensel and Biging, 1987), whereas in Europe they are only hesitatingly applied to actual forest management (Kolström, 1993; Nagel, 1996; Pretzsch and Kahn, 1996; Sterba and Monserud, 1998). This paper presents the basic equations, some application examples, and an evaluation approach of one of these models, which was developed at the Chair of Forest Yield Science in Munich, Germany (Pretzsch, 1992; Pretzsch and Kahn, 1996; Kahn and Pretzsch, 1997). The latest version includes the parameterization for Norway spruce (*Picea abies* (L.) Karst.), Silver fir (*Abies alba* Mill.), Scots pine (*Pinus sylvestris* L.), Common beech (*Fagus sylvatica* L.) and Sessile oak (*Quercus petraea* (Mattuschka) Liebl.), based mainly on empirical data from southern Germany. Further parameterization exist for Teak (*Tectona grandis* Linn. f.) stands in northern Thailand (Kahn, 1998) and Cork oak (*Quercus suber* L.) in Portugal. It has been applied for various forest management purposes by several German forest services and private forest owners as well as in research and as an instrument for educating forest students and forest managers.

In the following sections we distinguish the expressions 'model' and 'simulator'. Here, the term model is used equivalent to the set of equations that describe tree competition and growth. We will refer to the presented tool, however, as the growth simulator SILVA, which includes the implementation of the equations as a computer program, the input and output routines as well as the possibility to interactively influence a simulation run. Thus, stand development can be traced step by step during the simulation and thinning can be applied as a response to stand development. In many recent model developments (e.g. Wykoff et al., 1982; Wensel and Koehler, 1985; Pretzsch, 1992; Hasenauer, 1994; Hann et al., 1995; Sterba and Moser, 1995; Biber, 1996; Nagel, 1996; Crookston, 1997) such kind of modern user interfaces are implemented.

2. Database

The main data source for the parameterization of the model functions is the trial plot network, maintained by the Chair of Forest Yield Science. The data were gathered on 288 plots on 570 occasions, which means that some plots were repeatedly visited within the period between 1952 and 1998. These results include increment core data of newly established growth series from mixed stands at 10 different forest districts in Bavaria. In addition, in more than 1000 occasions (trial plot inventories), tree dimensions of all trees were measured, which are now available for model evaluation. The oldest of these inventories date back to the year 1870. All plots were treated in order to investigate certain forestry problems, with one of the most emphasized being the influence of thinning on tree development. Thus, particularly those trial plots that represent unthinned (A-variants) and differently thinned stands provide a unique data set for competition analyses.

Over all plots and inventories, 155,000 tree observations were available for model development. Around 30% of these data points were used for parameterization, including 15,549 data points for Norway spruce, 2040 for Silver fir, 14,210 for Scots pine, 13,180 for Common beech, and 3080 for Sessile oak. Within these trees, the range of diameters at breast height was between 5 and 93 cm for Norway spruce, 4.5 and 96 cm for Silver fir, 5 and 86 cm for Scots pine, 5 and 103 cm for Common beech and 7 and 90 cm for Sessile oak. The selected plots represent a wide variety of stand structures, which can be illustrated by the range of three structural indices: The 'Species Profile Index' (Pretzsch, 1996) varied between 0 (mono-layered) and 1.1 (strong vertical differentiation), the Clark–Evans index (Clark and Evans, 1954) ranged from 0.3 (extreme clustering) to 2.1 (regular spacing) and the Pielou segregation index (Pielou, 1974) was between 1 (pure stands)

Table 1
Stand characteristics of the data-pool used for parameterizing SILVA[a]

	hg (m)	dg (cm)	N (ha)	Ba (m^2/ha)	Vol (m^3/ha)	Amount
P. abies						
Tree measurements						15549
Plot inventories						151
Mean	24.7	29.9	911	23.59	295	
Max	43.1	59.6	9890	81.00	1430	
Min	5.1	5.5	1	0.20	2	
A. alba						
Tree measurements						2040
Plot inventories						81
Mean	27.1	38.6	73	8.0	112	
Max	45.1	67.0	294	28.5	393	
Min	8.1	8.5	3	0.2	1	
P. sylvestris						
Tree measurements						14201
Plot inventories						88
Mean	15.9	20.0	2947	17.8	121	
Max	37.2	60.1	17148	37.2	618	
Min	4.6	3.4	1	0.3	4	
F. sylvatica						
Tree measurements						13180
Plot inventories						183
Mean	24.5	28.1	326	12.7	175	
Max	41.5	60.3	2431	47.2	880	
Min	8.0	6.4	3	0.1	1	
Q. petraea						
Tree measurements						3080
Plot inventories						67
Mean	26.6	35.7	327	15.0	207	
Max	37.7	83.3	7233	41.0	616	
Min	9.0	5.0	28	10.4	10	

[a] hg and dg are stand height corresponding to the stem of average basal area; N, Ba and Vol are stem number, basal area and timber volume per hectare, respectively.

and −1 (rich admixture of tree species). An overview about the characteristics of the plot data is given in Table 1.

Additionally, data from the Forestry Research Station of Lower Saxony and the Swiss Research Station for Forest, Snow and Landscape in Switzerland were used for the development of the site-related potential height growth model (see Kahn, 1994).

3. Model description

In the following sections, tree representation, input and output routines of SILVA, as well as the equations of the core module, including the description of growth and mortality are described. Additionally, a flowchart of the simulator is presented in Fig. 1.

3.1. Tree representation

In SILVA, a tree is described by the following set of key variables: tree species, diameter at breast height (dbh), total height (h), height of crown base (hcb), crown diameter (cd), and tree coordinates (x, y). Every tree is assumed to stand straight upright. Species-specific crown models are used to represent three-dimensional crown shapes (Pretzsch, 1992). These models assume the crown to be rotation-symmetric

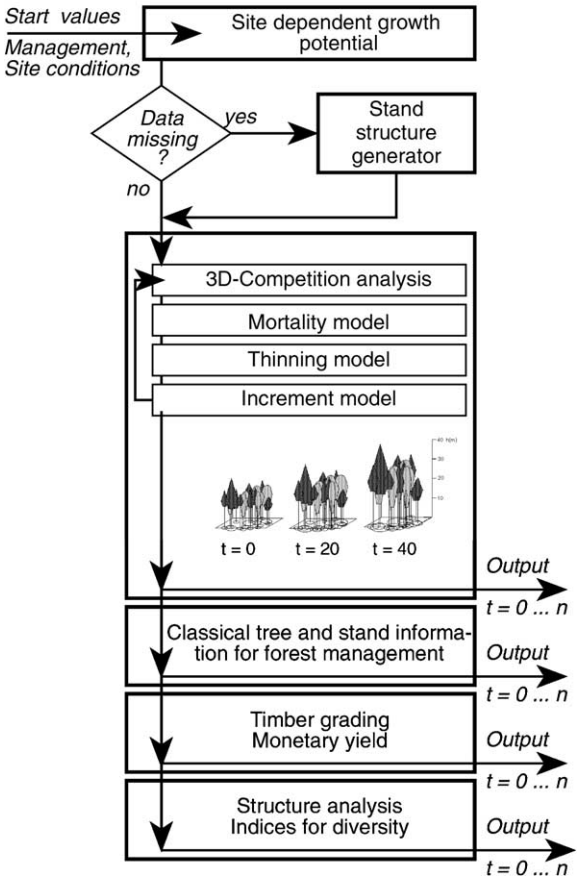

Fig. 1. Flowchart for a SILVA growth simulation. *t* means a point in time, *n* is the number of the respective year.

in horizontal direction and distinguish vertically between an upper and a lower part. Shape and relative length of upper and lower crown section is species-specific, but the crown is always assumed to be of maximum width (*cd*) at the height where both sections meet each other. For illustration, crown shapes for several tree species are presented in Fig. 2.

3.2. Initialization

3.2.1. Completion of single tree data

A simulation run is initialized with information about management, site conditions, and tree key variables. Especially, the latter is often incomplete and sometimes only the stem number and basal area per tree species are available. In such cases, the stand

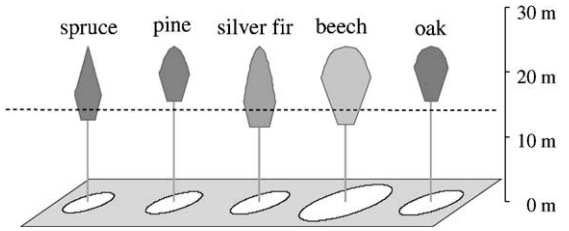

Fig. 2. Crown shape models for different tree species. Tree height 30 m, dbh = 30 cm.

structure generator module of SILVA (Biging et al., 1994; Pretzsch, 1997) can be used—automatically or interactively—to create single tree data that are in accordance to the given stand properties. Recently, the structure generator has been enlarged in order to derive representative tree data also from stratified sample plot measurements (Ďurský, 1999; Pommerening, 1999).

3.2.2. Management settings

Concept, frequency, and intensity of thinning define management conditions. SILVA allows, i.e. the representation of different types of selective thinning, future tree concepts, target diameter harvest, thinning from below or above, no thinning, and combinations of these concepts.

3.2.3. Site conditions

SILVA processes information from the following list of environmental conditions, given as long-term mean values:

- temperature difference between the coldest and the warmest month of the year (K),
- number of days per year with a mean temperature greater than 10 °C,
- mean temperature within the growing season (°C),
- relative soil water retention capacity (as coded by German site classification),
- precipitation sum within the growing season (mm).
- soil nutrient supply (as coded by German site classification),
- NO_x-concentration (ppm),
- atmospheric CO_2-concentration (ppm).

For most simulation purposes the environmental conditions are kept constant, but may also be altered during a simulation run if considered necessary

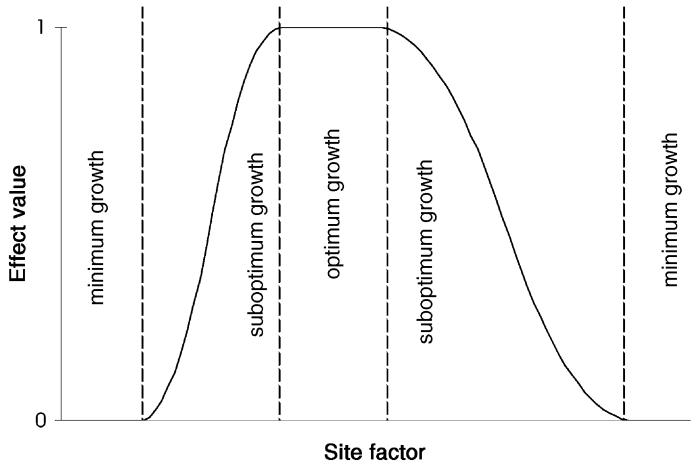

Fig. 3. Example of a response function as used in the site-height-potential submodel of SILVA.

(cf. Biber et al., 1998). Since the whole list of variables is seldom available for a single plot, it is possible to initialize most environmental conditions from the ecoregion code number (as defined by the German site classification), height above sea level, slope, and exposition. Climatic key data are then processed from a given list and altered according to the particular geographical conditions (Kahn, 1994). In this case, current concentrations of CO_2 and NO_x are used, and only soil and nutrient conditions have to be defined by the user.

Species-specific unimodal dose–response functions are used to compile the environmental information into values between 0 (minimum growth conditions) and 1 (ecological optimum of a tree species) (see Fig. 3 for illustration). These values are aggregated into ecologically significant site variables, considering also compensation effects (Kahn, 1994). Finally, these site variables determine the properties of a potential height growth curve, which is formulated according to the Chapman–Richards equation:

$$h_{pot} = A(1 - e^{-kt})^p \quad (1)$$

where h_{pot} is the potential tree height at age t and A, k, p the species-specific parameters, which are derived from a vector of site variables.

3.3. Core model description

3.3.1. General considerations

The smallest simulation time step with SILVA is a forest growth period of 5 years. This time interval corresponds with the time intervals provided by yield tables. It is also the standard time interval between two measurements on the trial plots used for model evaluation. The first step of each cycle is the three-dimensional competition analysis, which determines the degree of competition for each tree according to a number of indices described below. Then, a preliminary tree growth is determined that is used in the mortality module to decide if the individual is considered alive for the current simulation period. After the removal of dead or harvested trees, the competition indices and the dimensional changes of each tree are recalculated.

3.3.2. Inter-tree competition

One of the key features of single tree growth-simulators is the calculation of inter-tree competition of each tree, expressed as a competition index (cf. Biging and Dobbertin, 1992). The competition index *KKL* that is used in SILVA had been defined by Pretzsch (1995) and was evaluated by Bachmann (1998). It is calculated in two steps, including the determination of competitor trees and the actual determination of relative competition.

The competitors of a particular tree are identified from the application of a virtual reverse cone (Fig. 4). The axis of this cone is equal to the tree axis and its vertex is placed within the crown of the tree. The relative height within the crown and the angle of the vertex are species-specific. Any tree, whose top is inside this virtual cone, is regarded as a competitor.

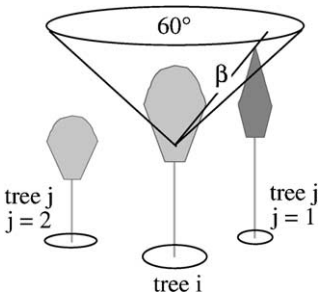

Fig. 4. Determining competitors for calculating the competition index KKL.

For any competitor the angle β between the insertion point of the cone and the top of the competitor tree is determined (Fig. 4). This angle is weighted by the relation between the crown cross-sectional areas (*CCA*) of the competitor and the tree of interest. These areas are calculated according to the respective crown models (see Pretzsch, 1992 for parameterization) either in the height of the cone vertex, if the cone vertex is above the maximum crown width, or in the height of maximum crown width, if the vertex is below. In addition, the angle β is also multiplied by a species-specific light transmission coefficient according to Ellenberg (1996). The competition index is defined as the sum of all competitor contributions:

$$KKL_i = \sum_{j=1}^{n} \beta_j \frac{CCA_j}{CCA_i} TM_j \tag{2}$$

where KKL_i is the competition index for tree i, β_j the angle between cone vertex and top of competitor j, CCA_j, CCA_i the crown cross-sectional area of trees j and i, respectively, TM_j the species-specific light transmission coefficient for tree j, n the number of competitors of tree i.

Besides *KKL*, SILVA uses two additional parameters for quantifying aspects of competition (Eqs. (7) and (10)). Since it was found that height and diameter growth of spruce, pine and beech is significantly affected by the type of competitor, and that this effect is not covered by the light transmission coefficients by Ellenberg (1996), an additional differentiation between deciduous and coniferous trees is introduced (*KMA*). *KMA* is a tree-type specific value that is calculated as the ratio between the sum of crown surface area of the coniferous competitors in relation to that of all competitors.

In addition to *KKL* and *KMA*, the influence of asymmetric spatial distribution of the competitor trees is considered by the variable *NDIST*, which is a measure for the horizontal distance of the center of gravity of competition from the stem center of the subject tree. Plot edge effects are corrected by linear expansion (Martin et al., 1977).

3.3.3. Mortality

After the degree of competition is determined, natural mortality within the next simulation cycle is estimated (Ďurský, 1997). For simplicity, dying trees are removed at the beginning of this cycle. The mortality model calculates the survival probability from the dimension of a tree and its expected basal area increase. The first step is the calculation of the dimensionless value P by means of a Logit-function to handle this dichotomous problem:

$$P = \frac{1}{1 + e^{-(a_0 + a_1 d + a_2(ig/dbh) + a_3(h/dbh) + a_4 SI)}} \tag{3}$$

where P is the dimensionless value used in Eq. (4) to determine mortality (threshold 0.5); ig the estimated tree basal area growth (cm^2/5 years); *SI* the site index, expressed as stand top height at age 50 years, estimated from site dependent height growth potential; $a_0, \ldots, a_4$ the estimated coefficients (see Appendix A).

The coefficients were estimated with the maximum likelihood method, using the properties of trees that died naturally and those of an equally sized random sample of surviving trees. By comparing equal numbers of dying and surviving trees, a model bias was avoided. In principle, all trees with P greater than a threshold of 0.5 could thus be classified as dying trees. However, since the residuals of the P-function are not homogeneously distributed, they had been used to calibrate the following function.

$$P_m = \frac{b_0}{e^{b_1 P^{b_2}}} \tag{4}$$

where P_m is the probability value of single tree mortality, $b_0, \ldots, b_2$ the tree species-specific estimated parameters (see Appendix A).

Finally, the value P_m determines the probability of single tree mortality. A particular tree is considered dead, if the value P_m is greater than a random number drawn with equal probability from the interval [0; 1].

3.3.4. Thinning

Thinning will be performed according to the settings defined by the user at the start of the simulation. Firstly, it is checked if the stand is inside the user-specified time or top height phase where thinning occurs. If this is the case, current stand density (basal area or stem number per hectare) is compared to the user-defined density threshold at a given top height of the stand. Tree removal is then executed according to the chosen thinning-concept until stand density drops below the given threshold value. The selection of thinning algorithms represents a great number of practice relevant thinning types. Some of them are based on a fuzzy logic controller (Kahn, 1995), whereas others use the *A*-value concept (Johann, 1983) to define the degree to which selection trees are released.

3.3.5. Height growth

Firstly, site-dependent height growth potential is calculated for each tree according to Eq. (1). Inserting the current tree height h_t and solving the equation for t, a theoretical age for this tree is obtained that represents the minimum number of years required to reach h_t (Pretzsch, 1992):

$$t = \frac{-\ln\left(\sqrt[p]{\frac{h_t}{A}}\right)}{k} \tag{5}$$

where t is the theoretical tree age (years), h_t the current tree height at theoretical tree age t (m).

Secondly, the simulation time step Δt (5 years) is added to t and inserted into the potential height growth estimation (Eq. (1)). Thus, the potential tree height $(h_{pot\,t+\Delta t})$ after the next simulation step is obtained. The potential height growth zh_{pot} can then easily be derived from the difference between $h_{pot\,t+\Delta t}$ and h_t (Monserud, 1975):

$$zh_{pot} = h_{pot\,t+\Delta t} - h_t \tag{6}$$

This potential height growth is then reduced to the expected height growth zh according to the individual conditions of a tree, which are defined by its competition indices and crown dimensions:

$$zh = zh_{pot} \cdot c_5 \cdot (1 - \mathrm{e}^{-c_0\,CSA}) \cdot \mathrm{e}^{-c_4\cdot(1+KMA)^{c_1}\cdot(1+NDIST)^{c_2}\cdot(KKL+c_3\Delta KKL)} \tag{7}$$

where zh is the expected height growth (m/5 years), *CSA* the crown surface area (m^2), $c_0, \ldots, c_5$ the species-specific parameters (see Appendix A).

zh is varied with a random error according to the distribution of residuals that was found during the calibration process of the model. This error consists of a species-specific fraction that represents unpredictable influences on growth at the individual level, and a second component that accounts for climatic variations between the simulation periods and which influences all trees in the same way. A correlation between the simulation periods is not considered. Finally, the height of a tree at the end of the time step is obtained by adding zh to h.

3.3.6. Diameter growth

Similar to the height growth model, tree diameter increase is also derived from a potential growth, which depends, however, not directly on site conditions. Firstly, potential diameter increment is calculated in dependence on tree diameter *dbh*:

$$zd_{pot} = j_1 \cdot (1 - \mathrm{e}^{-j_2\cdot dbh})^{j_3} \cdot j_2 \cdot j_3 \cdot \mathrm{e}^{-j_2\cdot dbh} \tag{8}$$

where zd_{pot} is the potential diameter increment (cm/5 years); j_1, j_2, j_3 the species-specific parameters (see Appendix A).

For further calculations, the potential basal area increment zg_{pot} is calculated from zd_{pot}:

$$zg_{pot} = [(dbh + zd_{pot})^2 - dbh^2]\frac{\pi}{4} \tag{9}$$

zg_{pot} is reduced to the expected basal area growth zg in a very similar way as the increase in height is derived from potential height growth:

$$zg = zg_{pot} \cdot ESto \cdot k_5 \cdot (1 - \mathrm{e}^{-k_0\,CSA}) \cdot \mathrm{e}^{-k_4\cdot(1+KMA)^{k_1}\cdot(1+NDIST)^{k_2}\cdot(KKL+k_3\cdot\Delta KKL)} \tag{10}$$

where $k_0, \ldots, k_3$ is the species-specific parameters (see Appendix A).

In this equation, the variable *ESto* is introduced, which represents the effect of site conditions on basal area growth. *ESto* is a species-specific function with soil nutrient supply, atmospheric CO_2- and NO_x-concentration as independent variables. As in the height growth calculation, residual variation is considered as a random modification of zg. Adding zg to the initial basal area of the subject tree results in basal area at the end of the simulation step, which can be easily transformed into the new dbh.

3.3.7. Development of crown dimensions

The first step in the calculation of the new crown dimensions is the estimation of height to crown base from height and diameter:

$$hcb = h(1 - e^{-(l_0 + l_1(h/dbh) + l_2\, dbh)}) \quad (11)$$

where $l_0, \ldots, l_2$ is the species-specific function parameters (see Appendix A).

Crown diameter is derived from height and diameter using the following exponential relationship:

$$cd = e^{m_0 + m_1 \ln(dbh) + m_2 h + m_3 \ln(h/dbh)} \quad (12)$$

where $m_0, \ldots, m_3$ is the species-specific function parameter (see Appendix A).

The data used for fitting the equation indicate that the presented model gives a better description of crown diameter than estimating cd from dbh alone (cf. Gill et al., 2000), especially in uneven aged mixed forests.

3.4. Output

From the simulated tree dimensions various aspects of stand development can be visualized within SILVA. For illustration of the three-dimensional stand development, perspective stand views and crown charts are provided, which are supplemented by a realistic stand visualization system (Pretzsch and Seifert, 1999) allowing virtual walkthroughs and interactive thinning. In addition, a large set of numerical information is available which can be viewed as diagrams or as text files that are similar to standard yield tables.

Three kinds of output can be distinguished: Firstly, classical growth and yield data are provided on a stand and tree level, e.g. stem number, basal area, timber volume, current and mean annual increment, and mean height. Secondly, monetary values can be obtained from the calculation of stand assortment distributions with the timber grading routine BDAT (Kublin and Scharnagl, 1988). This includes also detailed information about the monetary development of the stand, which is based on timber prices and harvesting costs specified by the user. If different prices and costs should be investigated, various economic scenarios can be evaluated with the same simulation run. The third group of output information describes ecologically important values. Several indices are calculated from tree dimensions and stand structure (e.g. Shannon, 1948; Clark and Evans, 1954; Penttinen et al., 1992; Stoyan and Stoyan, 1992; Pretzsch, 1996; Biber et al., 1998), which can be used to judge non-monetary values like habitat suitability or social forest functions.

4. Model application

4.1. Forest management

SILVA can be used in forest management for operational and strategic planning. For operational purposes management plans or timber production prognosis can be derived for single stands, forest enterprises, and landscape units (see Pretzsch et al., 1998). In general, this is executed on a short or medium-term perspective. In strategic planning, which normally covers longer time periods, SILVA helps to develop management guidelines for certain tree species or stand types under particular site conditions.

In the following, a simple example of strategic decision support is given. Timber production and economic yield are compared for different spruce-management concepts under typical site conditions in southern Germany. Different kinds of thinning are compared in terms of timber production and monetary yield. The risks associated with these management types are not considered in this example. The stand development is simulated from age 16 until 126 years. The following thinning strategies are defined:

- No thinning.
- *Combined thinning*. Up to a top height of 21 m the stand is thinned from above followed by a thinning from below at greater heights. The thinning intensity is strong, which is typical for the current local practice.
- *Future-tree concept*. 250 trees are selected, which are released by two to three competitor trees (on average 2.5 competitors per tree) at stand top heights of 10, 14, 18 and 22 m, respectively.
- *Selective thinning*. A decreasing number of selected trees are moderately released, according to the *A*-value concept (Johann, 1983). The *A*-value applied is 6 and thinning takes place in top height intervals of 4 m.

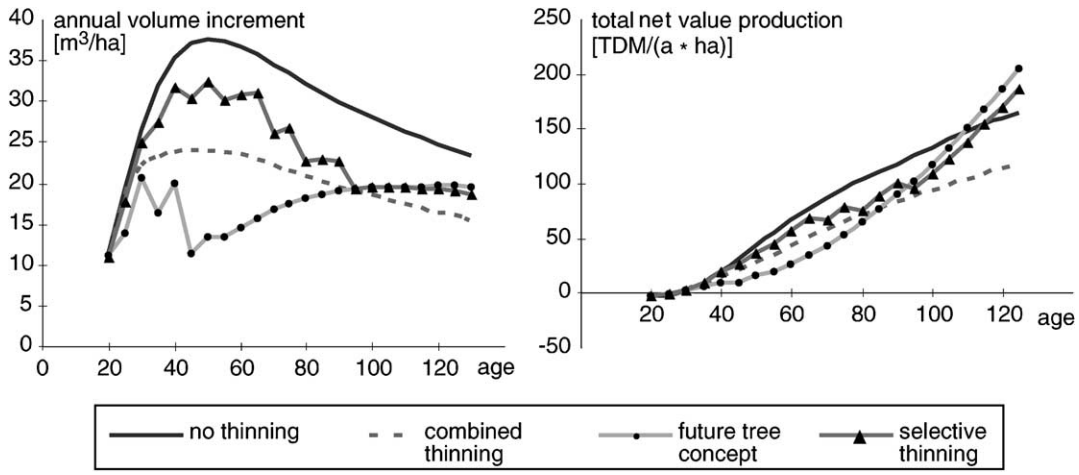

Fig. 5. Simulated annual volume increment (left) and total net value production (right) of differently treated Norway spruce stands.

Fig. 5 shows the results for each simulated treatment. Regarding the annual volume increment the no-thinning strategy is superior to all other strategies, with maximum growth rates of more than 35 $m^3\ ha^{-1}\ a^{-1}$ at age 50. The increment level with the combined thinning method is much smaller but results in the same shape of the curve, whereas the selective thinning results in an increment curve between the two previous concepts. The points in time, where the selective thinning is performed can easily be identified by the sawtooth-shaped parts of the curve. The future tree concept results in a completely different growth curve. The heavy thinning between age 20 and 50 constrain the volume increment under 20 $m^3\ ha^{-1}\ a^{-1}$, with a minimum around 10 $m^3\ ha^{-1}\ a^{-1}$ at age 50. However, after the last thinning, the volume increment recovers rapidly, overtops the increment level of the combined thinning at age 100, and thereafter approximately equals the production level of the selective thinning. If forest management primarily aims at high timber amounts within short production periods, the no-thinning-concept is thus favorable, followed by the selective thinning if longer production periods are taken into consideration.

Considering the total net value production (TNVP), meaning the net value of the growing stock and that of all previously harvested trees, differences between the concepts become apparent at age 30. The unthinned stand shows the highest TNVP until a stand age of 100 years. Thereafter, it is increasingly exceeded by the selective thinning and even more by the future tree concept. Thus, no-thinning or future tree selection seems to be the most advantageous strategy with respect to TNVP depending on the length of the rotation period. If risk management is additionally considered, selective thinning is possibly a good compromise.

4.2. Research

Considering forest research, SILVA is intended to investigate tree and stand responses to changing environmental conditions, including those induced by silvicultural management. This is possible because the traditionally gained knowledge from long-term experimental plots has been aggregated into the simulator and can thus be used to assess any combination of conditions that is within the range of the implemented information. Since this allows also for new combinations, the implementation of experimental plots can concentrate on the investigation of actually new conditions. With this respect, SILVA can also serve to define the requirements for new experiments.

SILVA was already applied for certain aspects of climate impact research (Biber et al., 1998; Pretzsch and Utschig, 1998; Pretzsch et al., 2000). This is illustrated with an example that shows the development of mean annual volume increment (MAI) of pure

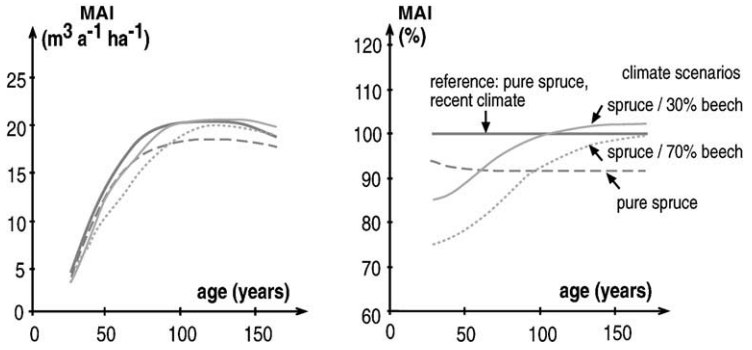

Fig. 6. Simulated MAI of pure spruce and spruce–beech mixed stands under recent climate conditions (left). Predicted MAI of pure spruce and spruce–beech mixed stands under warmer and dryer conditions relative to the MAI of pure spruce under recent climate (right).

spruce and spruce–beech mixed stands in central Bavaria under changed climatic conditions. Two scenarios were applied: In the first, the recent conditions were kept constant. In the second, the mean temperature in the vegetation period was increased by 2 °C, the duration of annual vegetation period was increased by 10 days, and the precipitation in the vegetation period was decreased by 10%. This scenario was defined in accordance to Fabian (1991). The simulation results are shown in Fig. 6.

On the left side of Fig. 6 the development of MAI is shown for different types of forests under recent conditions. The volume increment is in all cases around 20 $m^3\ ha^{-1}\ a^{-1}$ with a maximum around age 100 for spruce and 140 for beech, and slightly decreasing values after culmination. The growth pattern of the pure spruce stand shown in this diagram is used as the 100% reference line in the right section of Fig. 6 to illustrate the growth difference under changed climatic conditions. The resulting MAI for pure spruce stands is about 10% lower as simulated under current conditions. The estimated beech growth, however, is increased under the changed conditions and is thus able to compensate the growth losses of spruce in mixed stands to some degree. Because the growth culmination is different between the species, the degree of the compensation as well as the necessary time period for its establishment depends on the mixture. An admixture of 30% beech results in a higher increment at age 100, even higher compared to the current climate spruce response. An admixture of 70% beech has the same effect but the compensation point is reached approximately 50 years later.

4.3. Education

SILVA can be used for training of non-professional private forest owners, forest managers, and students, as well as to support public relations. For example, thinning can be executed and its effect can be impressively demonstrated with the simulator's stand visualization system (Pretzsch and Seifert, 1999). Trees can be marked by the user to be selected for thinning or as future trees, respectively (Fig. 7) and the development of the stand is shown as consequence of the management activities.

Additionally, SILVA is useful for the demonstration of biological and ecological principles of forest dynamics. For example the growth of light demanding and shade tolerant tree species as well as the dynamics of differently structured stands can be followed and compared over time.

5. Model evaluation

Evaluation is a very important recurring feature in the continuos process of developing a simulation tool (cf. Vanclay and Skovsgard, 1997). The term 'evaluation' refers to the necessary control of success and efficiency of a model with respect to a certain intention. In the following sections, we will shortly examine SILVA focussing on its suitability as a growth

Fig. 7. Aspect of a simulated forest stand generated by the visualization module of SILVA (Pretzsch and Seifert, 1999). The tree in the center of the picture is marked as a selection tree, three competitors have been marked for removal.

simulator, the validation of the underlying model, and the applicability of the implementation.

5.1. Suitability

The basic question is if the simulator is suitable to be integrated into the information flow of forest practice, which includes that the available pool of knowledge is used for efficient planning, execution and control of forest management. This is ascertained in SILVA particularly by its input and output structure. The initialization procedures provide the possibility for simulations even with incomplete data and allow the use of site information at very different resolution levels. The output methods and tools are developed in close contact with forest managers and thus are designed to meet their information demands. In comparison to classical yield tables, SILVA provides additional information that allows the estimation of monetary yield as well as the characterization of stand structure and—to a certain degree—biodiversity.

Considering the application on a wide range of currently observed site conditions, it is also important if the model approach represents tree growth responses in a biologically sound manner. Therefore, the variation of the height growth potential with site conditions in SILVA is based on the most important environmental variables that could be implemented from investigations of a large number of research plots. A site-sensitive single tree model is probably the only approach currently available that is able to utilize this data pool.

Finally, suitability implies that the degree of model complexity is in balance with the addressed task. In this context, particularly the single tree-approach could be questioned. However, single tree-based

representation of thinning is necessary to mirror modern thinning concepts that concentrate on a subset rather than on the whole population of trees in a stand. The concentration on single tree-development is furthermore consistent with the biological base unit that is actually investigated in the field.

5.2. Validation

The most important aspect of model validation is the comparison of prognosis and empirically observed growth. Permanent sample inventory plots render an independent data pool for this purpose, provided, they were not used to parameterize the model. However, since their observation period is rather short in most cases, the available long-term research plots are particular important for evaluation purposes. Quantitative validation relies on three basic features (Akça, 1997): The bias, which describes the mean difference between prognosis and observation, the precision value, expressed as standard deviation of absolute or relative differences between observed and simulated values, and the accuracy of the model, which means the degree of convergence of prognosis to observations. Accuracy may be small if the bias is high or the precision is low. A validation example on stand level is presented in Table 2 with data from 615 research plot inventories, which were taken between 1870 and 1995 and which were not used for the parameterization of the model. Because these inventories provide only sums and mean values of stand data, the missing data were generated with the stand structure generator of SILVA. For each inventory, a 5-year growth prognosis has been executed and simulated volume increment (iv) is compared with the respective observation. The relative bias of the simulated mean increment indicates a maximum overestimation of 1.9% (Norway spruce) and an underestimation of 4.8% at the most (Sessile oak). For Oak the best relative precision is obtained with 18.5% deviation of the mean observed increment, while pine shows the lowest precision with a respective value of 38.6%. Because the bias for all tree species is low, accuracy does not differ considerably from the precision.

Table 2
Example for model validation on stand level with respect to stand timber volume increment (VI)[a]

Species	*N*	Bias (%)	Precision (%)	Accuracy (%)
P. abies	220	−1.9	19.84	19.93
P. sylvestris	115	0.6	38.62	38.62
F. sylvatica	194	−0.7	28.98	28.99
Q. petraea	86	4.8	18.54	19.15

[a] *N*: number of research plot inventories; bias (%): mean deviation of observed and simulated VI relative to observed; precision (%): standard deviation of differences between observed and simulated relative to observed values; accuracy (%): square root of squared bias (%) plus squared precision (%).

Additionally, the qualitative aspects of validation can be examined. Therefore, it is investigated if the model behavior is in accordance with biological knowledge and practical experience concerning forest stand- and tree growth-dynamics. With this respect, SILVA has been shown to be in accordance with Assmann's concept of optimum basal area (Assmann, 1961), and the results of Kennel (1965a,b) as well as Magin (1959) according to the structure-related growth of mixed stands. For qualitative validation see also Biber (1996).

As an example, stand development as represented with SILVA is compared to the maximum density rule, published by Reineke (1933), which is widely accepted to express a biological principle. According to this rule, the natural logarithm of stem number per unit area can be expressed as a linear function of the natural logarithm of basal area mean diameter in fully stocked stands. The slope of this line does not significantly differ from −1.605. From a biologically plausible single tree model it should be expected that this stand level principle could be reproduced without being explicitly implemented. To test this assumption with SILVA, the stand structure generator is used to initialize 15 different Norway spruce stands. These stands are divided into five groups with the same mean diameter each, reaching from 10 to 45 cm. Each group consists of a stand with low, medium and high stem density. The trees were arranged regularly. We chose the site conditions to be optimal for Norway spruce. The development of all 15 stands was simulated over 100 years without any thinning routine.

Fig. 8 shows the simulation results in terms of stem number plotted over mean diameter in a double logarithmic grid. It is apparent that the trajectories approach and then follow a virtual linear borderline, which seems to be the density limit, independent of the initial stand density. The slope of the borderline

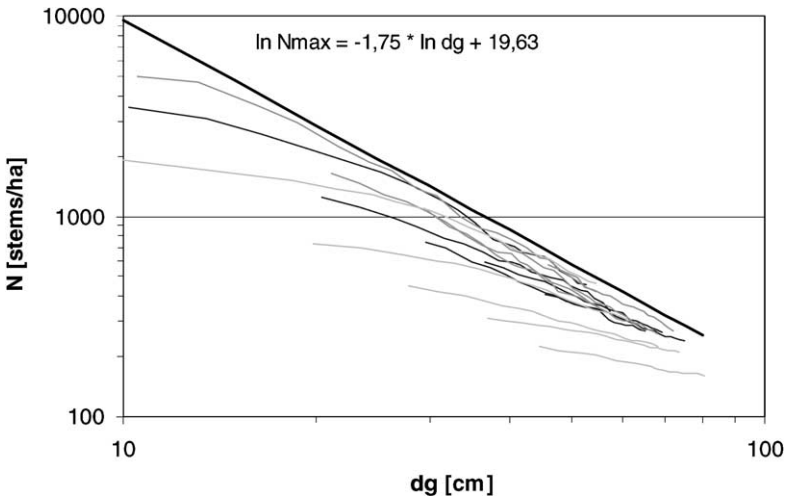

Fig. 8. Simulated development of non-thinned pure Norway spruce stands under equal site conditions but different starting dimensions and densities. The bold line indicates roughly the maximum stand density.

arbitrarily placed in Fig. 8 is −1.75, which is very close to the value postulated by Reineke.

5.3. Software characteristics for practical use

The way, in which a model is implemented in order to fulfill the needs of the actual user, is crucial for its acceptance. User friendly software design and documentation according to accepted conventions is most important. It is also favorable if the core module of a simulator is independent of the computer system, to enable an efficient exchange of modules between different research groups. Therefore, the SILVA core modules are currently transferred from Object Pascal programming language into C++. Input and output interfaces are designed in order to be appropriate for practical use from interactive operating at stand level to batch operation on enterprise level. Also the user's manual for SILVA (Seifert et al., unpublished) is edited according to the standards recently defined by the German Association of Forest Research Institutes (Deutscher Verband Forstlicher Forschungsanstalten, 2000).

6. Discussion

The current version of SILVA presented in this paper is a readily applicable tool for management, research and education purposes. As defined by its parameterization data, SILVA is reliable for most German site conditions, particularly those found in Southern Germany. The model equations are biologically meaningful and the parameter values are derived from a large data set. Thus, predictions are reasonable qualitatively as well as quantitatively and results are far more accurate than currently used standard yield tables (cf. Reimeier, 1999). It should be mentioned that the system of functions and parameters described represents only that part of SILVA that controls tree growth in relation to tree species, site and individual competitive situation. Additional equations and parameters (not listed in Appendix A) are used inside and outside this core. For example, several optional computations are available to substitute incomplete input information. This applies to environmental variables that could be computed from rough site information (e.g. German standard ecoregion codes) as well as for tree data, which are generated from stand or trial plot information if necessary (Pretzsch, 1997; Pommerening et al., 2000). Also, additional output routines can be used to calculate harvesting costs according to the German extended grades' tariff and to determine sales returns from wood price statistics (Pretzsch and Kahn, 1996). Furthermore, the timber grading module BDAT, which is implemented in SILVA to estimate stem volume, is itself a large set of statistically derived equations, developed by the Forestry Research Station

of Baden-Württemberg (Kublin and Scharnagl, 1988).

A very important feature, which distinguishes SILVA from most other practice relevant simulators is its site-related representation of potential height growth (Kahn, 1994). With this submodel, the traditional site index approach is replaced by an explanatory structure, which is very much similar to that of traditional gap models (e.g. Botkin et al., 1972; Prentice and Leemans, 1990). Thus, the flexibility of the model is considerably increased and allows the application of SILVA also for climate impact research as long as the climate scenarios do not exceed the climate conditions that are represented by the parameterization data too much. Although other model approaches may consider environmental conditions in higher resolution, climate impact-research with SILVA profits from its concentration on site data that are broadly available as well as its management options. Different management strategies can be assessed together with different environmental scenarios and appropriate mitigation strategies for unfavorable developments can be suggested (cf. Pretzsch and Utschig, 1998). Furthermore, possible changes of physiological properties to changing conditions, which are rather a problem in highly sophisticated models (e.g. Wang et al., 1996), are implicitly taken into account because the response functions are based on stands, which are acclimated to their particular site conditions. However, the drawback of the comparatively rough representation of environment, particularly the use of a 5-year time step, is that only mean value-changes but no shifts within a year and no damages from particular extreme events are considered (Grote et al., 1998).

In contrast to yield tables, a newly applied management practice can be easily represented, especially if stand structure or mixture changes within a rotation period. Another advantage above those types of models is the easy adaptation of parameters in SILVA to an enlarged data set, when new measurements from sample inventories or research plots become available. For example, a number of new long-term research plots have been recently established in Bavaria that represent different competition constellations in order to gain information about growth responses to also extreme releasement (Biber, 1996), which is currently under-represented in the database used for parameterization of SILVA. This possibility of reparameterization is particularly necessary if environmental conditions develop in a way never before experienced at any site (e.g. atmospheric CO_2-increase). Thus, it provides a chance to keep pace with the growth dynamics of forests (Spiecker et al., 1996).

Besides economical reasons, ecological aspects are getting more important since the prominent concept of sustainability has been enlarged from mere wood production to the whole set of forest functions. In order to fulfill the demands of this enlarged view, information about the ecological significance of a stand is required (Pretzsch, 2001). Some of this information is accessible in SILVA, because it includes explicit information about stand structure and species abundance. Thus, structural indices (Clark and Evans, 1954; Pielou, 1974; Pretzsch, 1996, 1997) and also more detailed spatial statistics (Penttinen et al., 1992; Biber and Weyerhaeuser, 1998) are compiled in order to provide a relatively simple but significant ecological characterization of forests. Based on these kinds of statistics, ecological analysis of different forest types with respect to habitat conditions had already been successful (Wiegand, 1998; Detsch, 1999), and stand development over time can be judged with respect to ecological consequences.

The ongoing development of SILVA includes a comprehensive evaluation process, which may even cause future changes in the existing model structure. Additionally, emphasis is put into the development of new submodels, e.g. for the simulation of natural regeneration or wood quality. Furthermore, new tree species, e.g. maple (*Acer pseudoplatanus* L.), ash (*Fraxinus excelsior* L.), and Douglas fir (*Pseudotsuga menziesii* (Mirb.) Franco) are going to be parameterized on the basis of newly established growth series in mixed and pure stands. Thus, we will further on address the objective to develop a reliable instrument for assistance in the decision-making process in forestry, which is applicable to a wide range of tree species and stand conditions.

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Appendix A.

Tables 3–8 contain the parameter values referring to the model equations shown above. They correspond to the current model version numbered SILVA 2.2. In general, the parameter values are given together with their standard errors. In some cases, however, parameter values were obtained by fitting preliminary grouped data. Thus, their degrees of determination and standard errors are not meaningful and are not shown here.

Table 3
Parameters of the mortality model (Eqs. (3) and (4))[a]

Tree species	Parameter	Value	Standard error
P. abies/*A. alba* ($N = 4764$)	a_0	5.3908	0.3301
	a_1	−0.0089	0.0027
	a_2	1.4802	0.0283
	a_3	−5.3998	0.2070
	a_4	−0.0406	0.0070
	b_0	99.400	–
	b_1	4.327	–
	b_2	1.628	–
P. sylvestris ($N = 6077$)	a_0	1.1889	0.2289
	a_1	0.0637	0.0107
	a_2	0.5770	0.0331
	a_3	−0.8963	0.1293
	a_4	−0.1142	0.0117
	b_0	99.637	–
	b_1	3.428	–
	b_2	0.664	–
F. sylvatica ($N = 526$)	a_0	6.6686	2.0206
	a_1	−0.2610	0.0658
	a_2	3.0796	0.2917
	a_3	−7.6495	1.5171
	a_4	0.2695	0.0596
	b_0	96.155	–
	b_1	3.593	–
	b_2	2.150	–
Q. petraea ($N = 1314$)	a_0	2.1001	1.1358
	a_1	−0.0488	0.0315
	a_2	0.4489	0.0758
	a_3	−2.5691	0.7416
	a_4	0.0461	0.0317
	b_0	100.195	–
	b_1	3.078	–
	b_2	0.3215	–

[a] The parameter a_1–a_4 were estimated by maximum likelihood method, parameters b_0–b_2 were obtained by fitting grouped data. N: sample size.

Table 4
Parameters of the height growth model (Eq. (7)) obtained by non-linear regression[a]

Tree species	Parameter	Value	Standard error
P. abies ($R^2 = 0.58, \text{MSE} = 0.10, N = 15\,549$)	c_0	0.1494	0.0016
	c_1	0.2236	0.0649
	c_2	0.1032	0.0442
	c_3	0.9068	0.0722
	c_4	0.1123	0.0051
	c_5	0.8233	0.0019
A. alba ($R^2 = 0.02$, $\text{MSE} = 0.18, N = 2040$)	c_0	2.5566	0.0000
	c_1	−0.0723	0.2775
	c_2	1.0112	0.1736
	c_3	−0.3986	0.0965
	c_4	0.1799	0.0318
	c_5	0.7434	0.0089
P. sylvestris ($R^2 = 0.52$, $\text{MSE} = 0.08, N = 14\,201$)	c_0	0.2898	0.0040
	c_1	0.9168	0.1823
	c_2	0	–
	c_3	−1.3423	0.2288
	c_4	0.0597	0.0078
	c_5	0.7686	0.0017
F. sylvatica ($R^2 = 0.52$, $\text{MSE} = 0.07, N = 13\,180$)	c_0	0.0220	0.0005
	c_1	−0.7276	0.0591
	c_2	0.3142	0.0602
	c_3	0	–
	c_4	0.1859	0.0047
	c_5	0.8543	0.0028
Q. petraea ($R^2 = 0.84$, $\text{MSE} = 0.03, N = 3080$)	c_0	0.1544	0.0089
	c_1	0	–
	c_2	0.5134	0.0879
	c_3	1.2968	0.2342
	c_4	0.0921	0.0040
	c_5	0.7848	0.0028

[a] N: sample size; R^2: degree of determination; MSE: mean square error.

Table 5
Parameters of the potential diameter growth model (Eq. (8)) (the parameter values were obtained by fitting grouped data)

Species	Parameter	Value
P. abies	j_1	3963.0628
	j_2	0.0142
	j_3	0.2583
A. alba	j_1	1381.1821
	j_2	0.0150
	j_3	0.7425
P. sylvestris	j_1	877.5452
	j_2	0.0287

Table 5 (*Continued*)

Species	Parameter	Value
	j_3	0.4714
F. sylvatica	j_1	1508.9183
	j_2	0.0164
	j_3	0.7752
Q. petraea	j_1	1508.9183
	j_2	0.0129
	J_3	0.5384

Table 6
Parameters of the diameter growth model (Eq. (10)) obtained by non-linear regression[a]

Species	Parameter	Value	Standard error
P. abies	k_0	0.1044	0.0041
($R^2 = 0.69$, MSE =	k_1	0.3042	0.0596
2380, $N = 15\,549$)	k_2	−0.1285	0.0373
	k_3	0.2855	0.0383
	k_4	0.5829	0.0251
	k_5	0.5057	0.0040
A. alba	k_0	6.9068	0.0000
($R^2 = 0.45$, MSE =	k_1	0.6881	0.6689
2983, $N = 2040$)	k_2	0.8425	0.3863
	k_3	−0.4709	0.1284
	k_4	0.3462	0.1475
	k_5	0.3550	0.0119
P. sylvestris	k_0	0.1133	0.0022
($R^2 = 0.68$, MSE =	k_1	4.3875	0.1457
262, $N = 14\,201$)	k_2	0.5017	0.0728
	k_3	−1.2123	0.1101
	k_4	0.0440	0.0053
	k_5	0.4676	0.0025
F. sylvatica	k_0	1.4931	0.0049
($R^2 = 0.74$, MSE =	k_1	0.1537	0.0806
2009, $N = 13\,180$)	k_2	0.7364	0.0000
	k_3	0	–
	k_4	0	–
	k_5	0.4914	0.0036
Q. petraea	k_0	10	–
($R^2 = 0.66$, MSE =	k_1	0	–
1605, $N = 3080$)	k_2	0.6741	0.1432
	k_3	0.4998	0.0321
	k_4	0	–
	k_5	0.5422	0.0048

[a] N: sample size; R^2: degree of determination; MSE: mean square error.

Table 7
Parameters of the height to crown base model (Eq. (11)) obtained by nonlinear regression[a]

Species	Parameter	Value	Standard error
P. abies	l_0	−0.0443	0.0315
($R^2 = 0.79$, MSE =	l_1	−0.8823	0.0241
6.31, $N = 8705$)	l_2	−0.0004	0.0003
A. alba	l_0	0.1409	0.0625
($R^2 = 0.64$, MSE =	l_1	−0.8480	0.0642
15.56, $N = 1898$)	l_2	−0.0042	0.0006
P. sylvestris	l_0	0.3760	0.0417
($R^2 = 0.93$, MSE =	l_1	−0.9963	0.0358
2.09, $N = 2584$)	l_2	−0.0218	0.0005
F. sylvatica	l_0	−0.5478	0.0229
($R^2 = 0.73$, MSE =	l_1	−0.1094	0.0158
10.67, $N = 14\,075$)	l_2	−0.0023	0.0003
Q. petraea	l_0	−0.9967	0.0334
($R^2 = 0.78$, MSE =	l_1	−0.2043	0.0252
4.67, $N = 6281$)	l_2	0.0032	0.0003

[a] N: sample size; R^2: degree of determination; MSE: mean square error.

Table 8
Parameters of the crown diameter model (Eq. (12)) obtained by non-linear regression[a]

Species	Parameter	Value	Standard error
P. abies	m_0	0.2195	0.0419
($R^2 = 0.73$, MSE =	m_1	0.2545	0.0201
0.73, $N = 8416$)	m_2	0.0090	0.0009
	m_3	−0.6735	0.0202
A. alba	m_0	0.1071	0.0918
($R^2 = 0.48$, MSE =	m_1	0.4506	0.0475
1.82, $N = 1726$)	m_2	0.0009	0.0026
	m_3	0.0975	0.0473
P. sylvestris	m_0	−0.5515	0.1093
($R^2 = 0.79$, MSE =	m_1	0.6468	0.0567
0.64, $N = 2804$)	m_2	−0.0062	0.0029
	m_3	0.1904	0.0601
F. sylvatica	m_0	0.8556	0.0483
($R^2 = 0.62$, MSE =	m_1	0.4299	0.0238
2.94, $N = 12\,733$)	m_2	−0.0035	0.0012
	m_3	−0.3238	0.0242
Q. petraea	m_0	0.3737	0.1159
($R^2 = 0.84$, MSE =	m_1	0.1168	0.0527
1.24, $N = 6436$)	m_2	0.0284	0.0022
	m_3	−0.9340	0.0525

[a] N: sample size; R^2: degree of determination; MSE: mean square error.

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